MATHEMATICS APPLICATIONS UNIT 3

Section One: Calculator-free

SOLUTIONS

WA student number: In figures ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐

In words   _____

_____

Your name   _____

Time allowed for this section

Reading time before commencing work: five minutes
Working time: fifty minutes

Number of additional answer booklets used (if applicable): ☐

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	7	7	50	52	35
Section Two: Calculator-assumed	12	12	100	98	65
				Total	100

Instructions to candidates

1.	The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2.	Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3.	You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4.	Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5.	It is recommended that you do not use pencil, except in diagrams.

6.	Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7.	The Formula sheet is not to be handed in with your Question/Answer booklet.

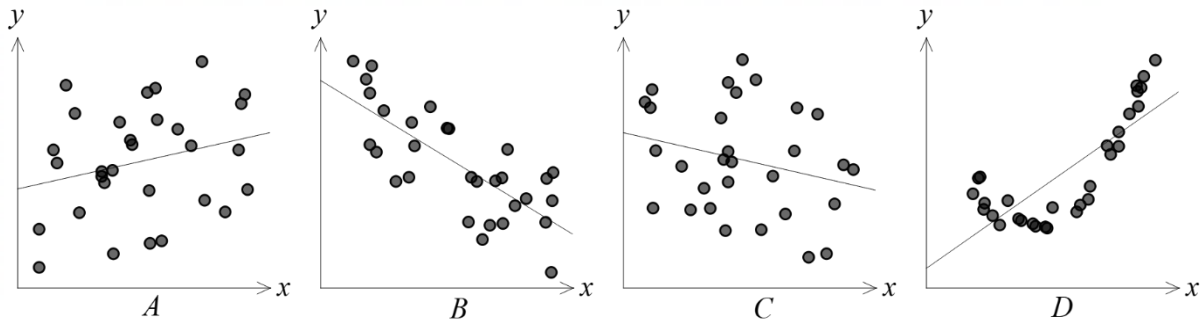   SN175-216-3

Section One: Calculator-free **35% (52 Marks)**

This section has **seven** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

See next page

Question 1   **(7 marks)**

Scatterplots for datasets A, B, C and D are shown below, together with the least-squares line for each dataset.

(a) Which dataset best illustrates the existence of a weak positive relationship between the variables? Justify your choice by referring to two key features of its scatterplot. (3 marks)

Solution
Dataset A – the least-squares line for its scatterplot has a positive slope and the points are widely spread around this line indicating a weak relationship.
Specific behaviours
✓ correct dataset
✓ indicates slope of line is positive
✓ indicates points are widely spread around line

(b) Which dataset best illustrates the existence of a non-linear relationship between the variables. Justify your choice by referring to at least one key feature of its scatterplot.

(2 marks)

Solution
Dataset D – the points are not in a straight line but instead closely follow a curve.
Specific behaviours
✓ correct dataset
✓ indicates points closely follow a curve

Let r_A, r_B, r_C and r_D be the values of the correlation coefficient between the variables for datasets A, B, C and D respectively.

(c) Use estimates for the coefficients to arrange r_A, r_B, r_C and r_D in ascending numerical order.

(2 marks)

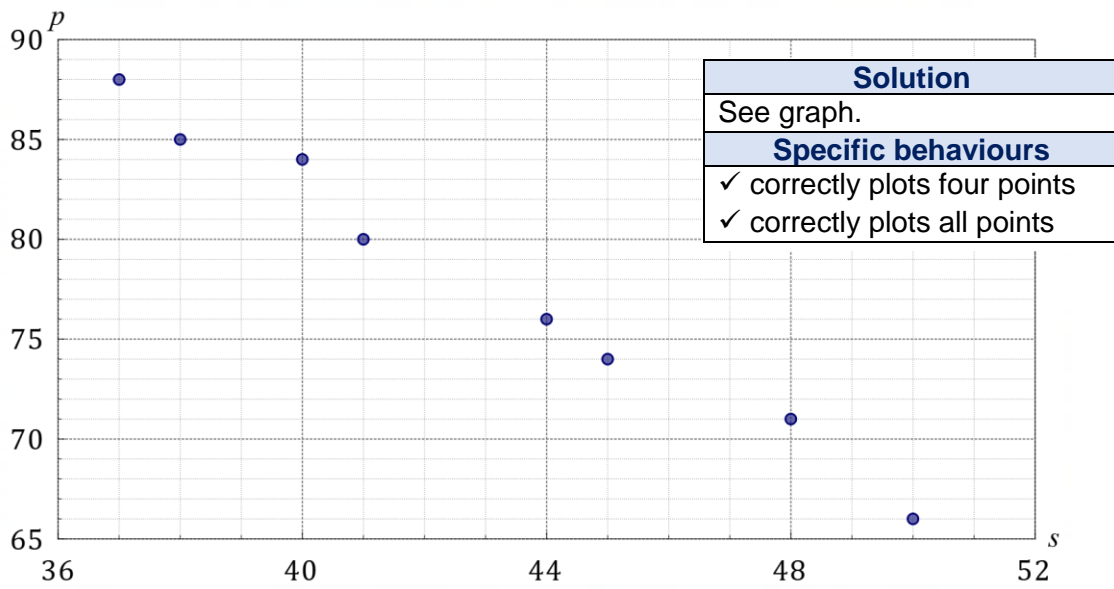| Solution |
| --- |
| $r_A \approx 0.2, r_B \approx -0.8, r_C \approx -0.2, r_D \approx 0.8$ |
| Hence ascending numerical order is r_B, r_C, r_A, r_D. |
| **Specific behaviours** |
| ✓ indicates reasonable estimates for at least 3 coefficients |
| ✓ correct order |

Question 2 (6 marks)

A research institute placed healthy coral in eight reef tanks for a three-week period. For each tank, the average salinity of the water over that time, s in parts per thousand, and the percentage p of coral that remained healthy at the end of the three weeks was recorded in the table below.

s	45	38	40	48	44	41	37	50
p	74	85	84	71	76	80	88	66

(a) Construct a scatterplot of the data on the axes below. (2 marks)

Solution
See graph.
Specific behaviours
✓ correctly plots four points
✓ correctly plots all points

(b) Comment on the validity of each of the following statements:

(i) Average salinity is a good predictor for the percentage of coral that will remain healthy after three weeks in a reef tank. (2 marks)

Solution
Statement is valid, as the scatterplot shows a strong association exists between the variables.
Specific behaviours
✓ indicates statement valid
✓ reasonable justification of why statement valid

(ii) Increasing the average salinity causes the percentage of coral that will remain healthy after three weeks in a reef tank to decrease. (2 marks)

Solution
Not a valid statement. The observed strong association between the variables does not necessarily mean that there is a causal relationship - the association may be due to a common response to another variable.
Specific behaviours
✓ indicates statement not valid
✓ reasonable justification of why statement not valid

Question 3 **(7 marks)**

The first term of a sequence is 1220 and the next three terms, in order, are $1300, 1380$ and 1460.

(a) Is this sequence arithmetic, geometric or neither? (1 mark)

Solution
Arithmetic.
Specific behaviours
✓ correct answer

(b) Determine a recurrence relation that defines this sequence. (2 marks)

Solution
$T_{n+1} = T_n + 80, \qquad T_1 = 1220$
Specific behaviours
✓ states recurrence relation
✓ states term of sequence

(c) Deduce a rule for the nth term of this sequence. (1 mark)

Solution
$T_n = 1220 + (n-1)(80)$
$= 1220 + 80n - 80$
$= 1140 + 80n$
Specific behaviours
✓ correct rule in form $T_n = a + (n-1)d$ or simplified

The terms of this sequence model the monthly balance of a savings account. This balance was $1220 during the first month.

(d) During which month will this balance first reach $2100 or more? (3 marks)

Solution
$1140 + 80n = 2100$
$80n = 960$
$8n = 96$
$4n = 48$
$n = 12$
During the 12th month.
Specific behaviours
✓ correctly uses rule from (c) to form equation
✓ shows two or more correct simplification steps
✓ states correct month
Also accept correct continuation of sequence as justification

Question 4 **(7 marks)**

Amit has drawn connected graph G in the plane to help solve a map colouring problem. Its edges are not directed, it has 4 faces and its adjacency matrix C is

$$C = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \end{bmatrix}$$

(a) Use one or more elements of C to explain why G

(i) is **not** a complete graph. (2 marks)

Solution
$c_{3,2} = 0$ means there is no edge between vertices 3 and 2, but to be a complete graph every vertex must be connected to all other vertices.

Specific behaviours
✓ refers to any zero in the graph not on the diagonal
✓ relates to property of complete graph

(ii) contains a bridge. (2 marks)

Solution
In the second row all elements are zero except for $c_{2,1}$, which means that vertex 2 is only connected to the rest of the graph with one edge, and so this edge is a bridge.

Specific behaviours
✓ refers to any row or column that contains a single 1
✓ infers that edge must be a bridge

(b) Show that G satisfies Euler's formula. (3 marks)

Solution
Euler's formula is $v + f - e = 2$, and $f = 4$ is given above.
e = sum of digits above diagonal in matrix = 7.
v = number of rows (or columns) in matrix = 5.
Hence $v + f - e = 5 + 4 - 7 = 2$.

Specific behaviours
✓ indicates correct number of edges
✓ indicates correct number of vertices
✓ correctly uses Euler's formula

Question 5   **(10 marks)**

Bart delivers bread from his depot at D to all 6 grocery stores located at A, B, C, E, F and G daily.

In the graph below the vertices represent the depot and shops, the edges represent direct roads between shops and the edge weights are the travel times along these roads in minutes.
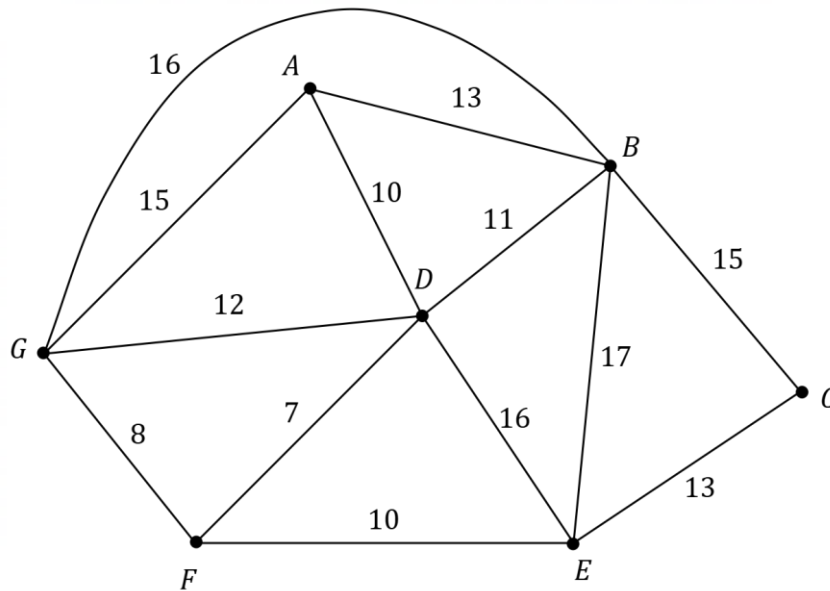
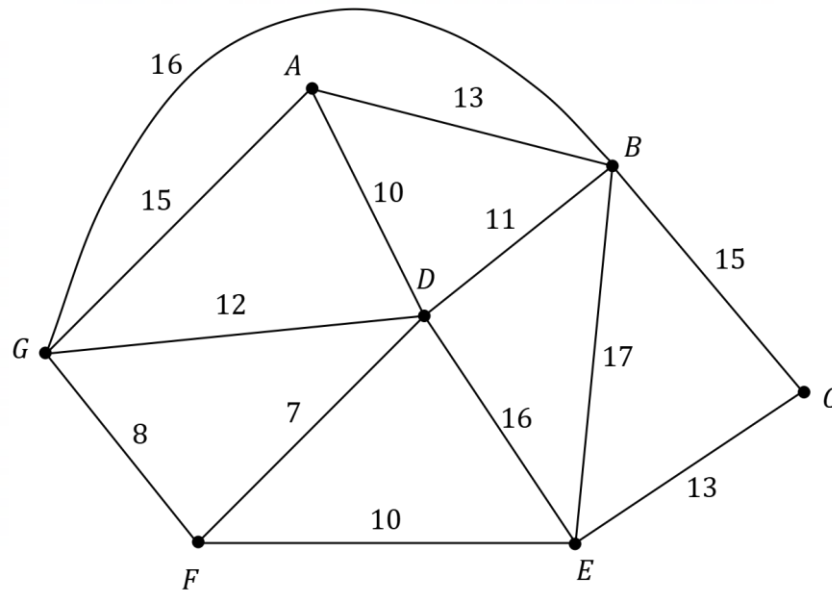(a)   Explain why the graph shown above is Hamiltonian.   (2 marks)

Solution
The graph contains a cycle (closed path) that includes all vertices once.

Specific behaviours
✓ states cycle or closed path
✓ states cycle includes all vertices once

Bart always starts and finishes a daily delivery from his depot, and never visits a store or travels along the same road more than once. He always spends 6 minutes at each store to complete the delivery.

(b)   One day, Bart left the depot at $6{:}45$ am and started his delivery round by travelling to shop E, as it had requested an urgent delivery. List the order that the shops received their deliveries and determine the time that Bart returned to the depot on this day.   (4 marks)

Solution
Delivery order from D: $E - C - B - A - G - F$.
Journey time: $16 + 13 + 15 + 13 + 15 + 8 + 7 = 87$
Time at shops: $6 \times 6 = 36$
Returns at $6{:}45 + 123 = 6{:}45 + 2{:}03 = 8{:}48$ am.

Specific behaviours
✓ correct order of deliveries
✓ indicates correct journey time
✓ correctly allows for time at each shop
✓ correct return time

See next page

(c) Determine the order in which Bart should visit the stores so that he returns to the depot in the least possible time, and state what this time is. Justify your answer. A copy of the graph has been provided below. (4 marks)
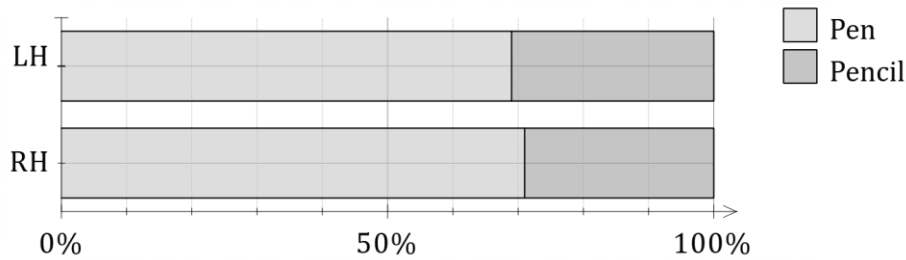

Solution
Order for shortest Hamilton cycle from D is:
$$A - B - C - E - F - G = 81$$
Least possible time: $81 + 36 = 117$ minutes.
Other possible Hamilton cycles from D with times are:
$$A - G - F - E - C - B = 82$$
$$F - E - C - B - A - G = 85$$
$$F - E - C - B - G - A = 86$$
NB All orders listed above may be reversed
Specific behaviours
✓ lists two or more alternative Hamilton cycles to part (b)
✓ correctly calculates one alternative journey time
✓ identifies shortest possible Hamilton cycle
✓ correctly states least possible journey time

Question 6   **(9 marks)**

(a) The following divided bar graph shows the writing preferences (pen or pencil) for a sample of left and right-handed students.

Explain whether the data indicates the presence of a clear association between dominant hand and writing preference.   (2 marks)

Solution
No suggestion of an association between the variables since close to 70% of both left- and right-handed students prefer pen.
Specific behaviours
✓ indicates no association
✓ explanation using similar percentages

(b) The table below shows the responses of people who house share in either Perth or Melbourne to a survey about their financial position pre-COVID to now.

	Financial position pre-COVID to now		
	Better	Same	Worse
Melbourne	34	100	66
Perth	10	30	10

(i) Calculate how many more people responded to the survey in Melbourne compared to Perth.   (2 marks)

Solution
$M = 34 + 100 + 66 = 200, \ P = 10 + 30 + 10 = 50.$ Hence $200 - 50 = 150$ more people.
Specific behaviours
✓ one correct city total
✓ correct difference

See next page

(ii)   Construct a row percentaged table for the data.   (3 marks)

Solution		
Melbourne figures are halved to obtain percentages.		
Perth figures are doubled to obtain percentages.		

	Financial position pre-COVID to now		
	Better	Same	Worse
Melbourne	**17**	**50**	**33**
Perth	**20**	**60**	**20**

Specific behaviours
✓ table with correct row and column headings
✓ figures in one row add to 100
✓ correct table

(iii) State, with reasons, whether the data suggests an association is present between the two variables.   (2 marks)

Solution
The data does suggest an association exists between the variables as a clear difference is observed in the percentages across the 'worse' category – 33% of those in Melbourne are worse off compared to only 20% of those in Perth.

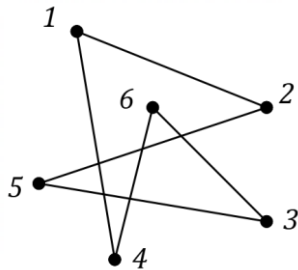| **Specific behaviours** |
| --- |
| ✓ indicates presence of an association |
| ✓ explanation using difference observed in percentages in columns |

Question 7 **(6 marks)**

For each of the following questions, answer yes or no and then justify your answer. Responses given without justification will not be awarded any marks.

(a) Is the graph shown below bipartite? (2 marks)

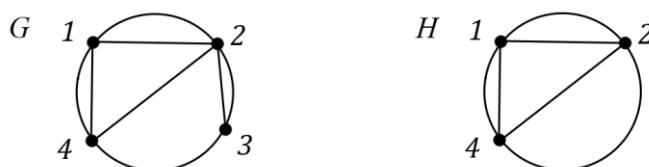| Solution |
| --- |
| Yes – the vertices can be split into two distinct groups: |
| **Specific behaviours** |
| ✓ answers yes |
| ✓ any correct justification |

(b) Is a connected planar graph with 3 vertices and 5 edges a simple graph? (2 marks)

Solution
No. A complete graph with 3 vertices (K_3, shown below) has 3 edges, and so the extra edges must be either be loops and/or multiple edges and a simple graph does not contain these.
Specific behaviours
✓ answers no
✓ justifies using properties of simple graph

(c) Is graph H a subgraph of graph G? (2 marks)

Solution
No. H has multiple edges between vertices 2 and 4 but G does not.
Specific behaviours
✓ answers no
✓ any correct justification

End of questions

Supplementary page

Question number: _____

Supplementary page

Question number: _____

Supplementary page

Question number: _____